Identifying Player Prototypes in Basketball using Archetypoid Analysis

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Team work

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Outline

Motivation

Motivation example

NBA Player Statistics Season 2024-25

$n = 501$ **players**

$p = 17$ **variables**

Var	Description
MIN	**Total minutes played**
PTS	**Points per game**
REB	**Rebounds per game**
AST	Assists per game
FGA	Field goal attempts
FG%	Field goal efficiency
P3A	3-point attempts
P3%	3-point efficiency
FTA	Free throw attempts
FT%	Free throw efficiency
STL	Steals per game
BLK	Blocks per game
TOV	Turnovers per game
PF	Personal fouls

Head of the NBA dataset (some variables)

Player	Team	PTS	FG%	P3%	REB	AST	BLK	MIN
A.J. Lawson	TOR	22.5	42.1	32.7	8.2	3.0	0.6	486
Aaron Gordon	DEN	24.4	53.1	43.6	8.1	5.3	0.5	1447
Aaron Holiday	HOU	20.1	43.7	39.8	4.6	4.9	0.7	792
Aaron Nesmith	IND	22.8	50.7	43.1	7.5	2.3	0.7	1122
Aaron Wiggins	OKC	24.7	48.8	38.3	8.0	3.6	0.5	1744
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- Can we found players who are **extreme** profiles based on their performance? Archetypoid analysis (**Standard ADA**).
- Are they players determined by ADA **representative**? e.g. do they have play enough minutes? Our proposal: **Penalized ADA**.

Background: AA & ADA

What are the Archetypes?

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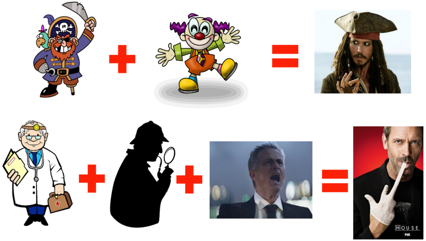
  - Jack Sparrow: 40% pirate and 60% clown.
  - Dr. House: 20% doctor, 30% detective, and 50% bad temper.



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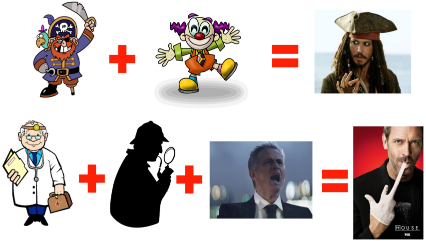

- In Statistics, the **concept of archetypes** is the same as in common life.

Archetype Analysis (AA). Definition

- **Objective** (Cutler and Breiman, 1994): to find a few, not necessarily observed, extremal cases or pure types (the archetypes) such that:
  - all the observations are **obtained by convex combinations of the archetypes**, and
  - all the archetypes are convex combinations of the observations.

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- **Solution** (Cutler and Breiman, 1994): Alternating minimizing algorithm and implemented in R by Eugster and Leisch (2009).

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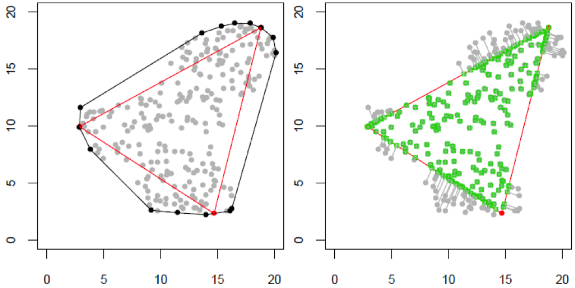

Figure 2: Visualization of three archetypes.

Suppose we want to find $k = 3$ archetypes:

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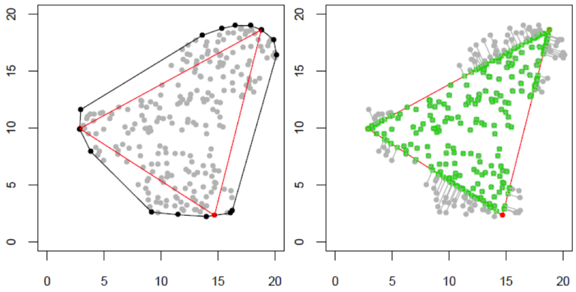

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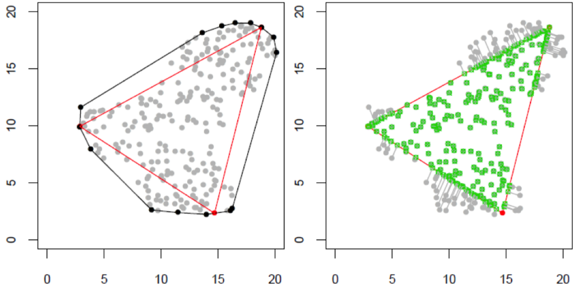

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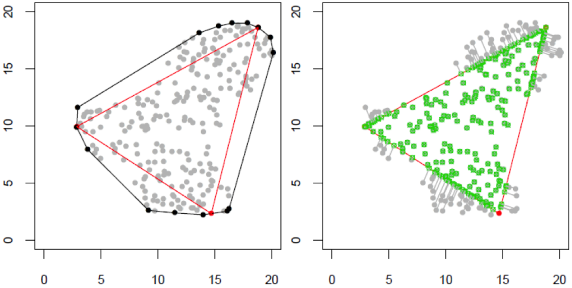

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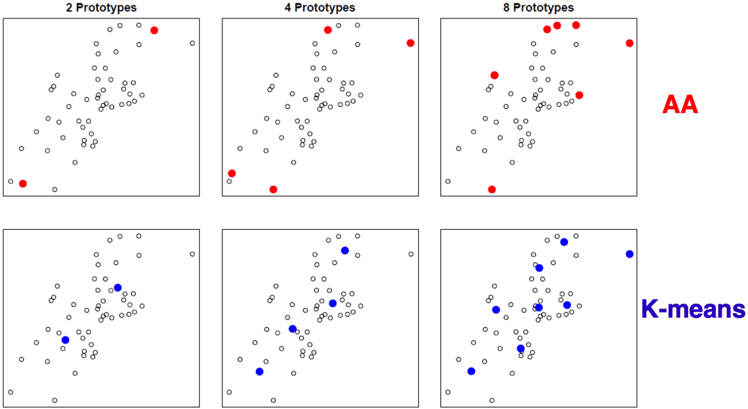
4. The **observations** are approximated by convex combinations of the archetypes.

Archetype Analysis (AA). Example

Archetypes **are more interpretable** than the centroids of k-means.



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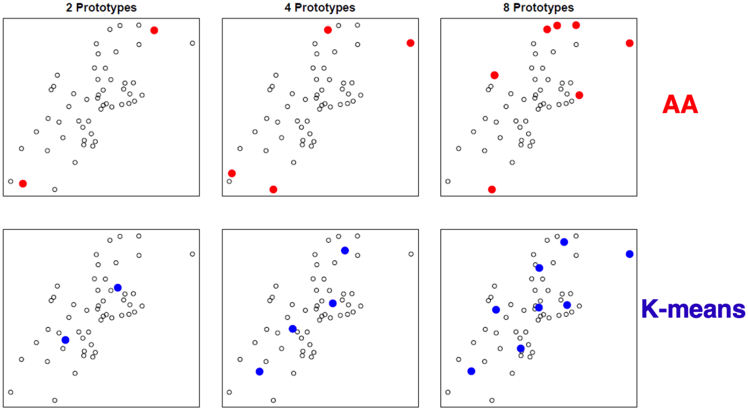

For humans, it is **easier to understand opposites than centroids**: they provide **an intuitive understanding** of the results (Vinúe *et al.*, 2015; Thurau *et al.*, 2012).

AA for multivariate data

- Let X be an $n \times m$ matrix with n observations and m variables.
- The objective of AA is to find the matrix Z of k m-dimensional archetypes.
- AA computes two matrices α and β which **minimize the residual sum of squares (RSS)**:

$$RSS = \sum_{i=1}^{n} \|\mathbf{x}_i - \sum_{j=1}^{k} \alpha_{ij}\mathbf{z}_j\|^2 = \sum_{i=1}^{n} \|\mathbf{x}_i - \sum_{j=1}^{k} \alpha_{ij} \sum_{l=1}^{n} \beta_{jl}\mathbf{x}_l\|^2,$$

under the constraints

1) $\sum_{j=1}^{k} \alpha_{ij} = 1$ with $\alpha_{ij} \geq 0$ for $i = 1, \ldots, n$ and

2) $\sum_{l=1}^{n} \beta_{jl} = 1$ with $\beta_{jl} \geq 0$ for $j = 1, \ldots, k$.

Archetypoid Analysis (ADA)

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- **Objective** (Vinúe, Epifanio, & Alemany, 2015): To find a few observed, **extremal cases or pure types (the archetypoids)** such that:
  - All observations are approximated by convex combinations of the archetypoids (**as before**), and
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- **ADA solution** (Vinué, Epifanio, & Alemany, 2015): algorithm based on PAM.

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- **Main idea**: Modify the **objective function**, explicitly controlling the selection via regularization (weights).

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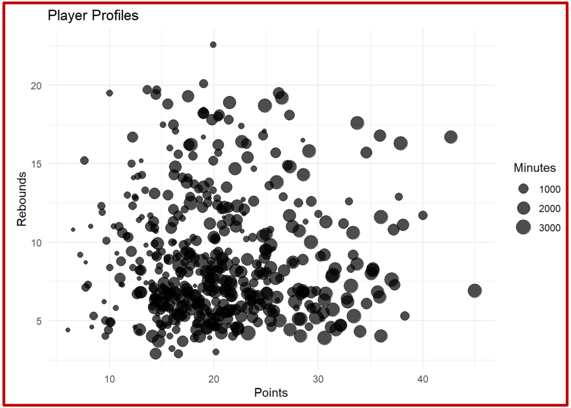
2. The **penalty weights** are defined as:
   - $w_\ell^0 = 1/(\mathrm{minutes}_\ell + \epsilon)$, where ϵ is a small constant to avoid division by zero,
   - $w_\ell = w_\ell^0 / \bar{w}^0$, where $\bar{w}^0$ is the mean of all the w_ℓ^0,
   - $\lambda \geq 0$ controls the strength of the penalty; $\lambda = 0$ represents no penalization.

- **Interpretation**: High-minute players $\rightarrow$ low contribution to the penalty

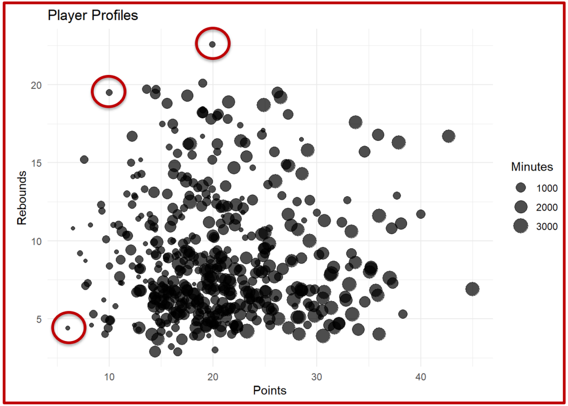
Application: NBA data set

ADA results (players)



- The variables **Points** & **Rebounds** are our features of interest.
- The variable **Minutes** serves to compute weights.

Standard ADA



- The variables **Points** & **Rebounds** are our features of interest.
- The variable **Minutes** serves to compute weights.
- Note that some players with very **low minutes** are located at the **extremes** of the data cloud $\Rightarrow$ Archetypoids in the **standard ADA**.

ADA results: Standard vs. Penalized (I)

- Example: $k = 3$ archetypoids, 20 starting points, & $\lambda = 1$ as the penalty strength.

	Standard	**Penalized**
RSS	0.027	0.159
Avg. minutes	988.97	2421.67
Computation time	27.95	41.11

- **RSS** is higher in the penalized version $\Rightarrow$ adding the penalty selects **less extreme** but more **representative** archetypoids.
- **Average minutes** nearly triples in the penalized version (989 $\rightarrow$ 2422 min) $\Rightarrow$ confirms the penalty favors **high-minute** players.

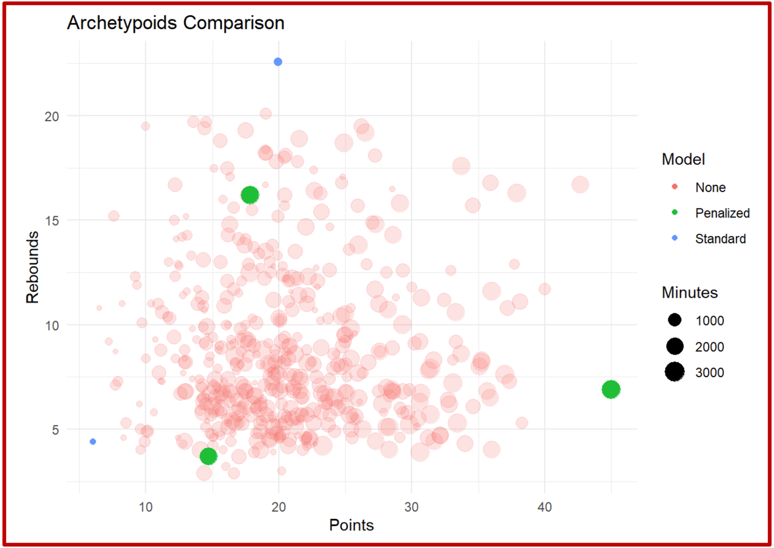
ADA results: Standard vs. Penalized (II)

Selected archetypoids

Player	PTS	REB	MIN	Model
Joe Ingles	6.0	4.4	114.1	Standard
Oscar Tshiebwe	19.9	22.6	255.2	Standard
Shai Gilgeous-Alexander	45.0	6.9	2597.6	**Both**
Rudy Gobert	17.8	16.2	2388.4	Penalized
Kentavious Caldwell-Pope	14.7	3.7	2279.0	Penalized

- Standard ADA picks players with **very low minutes** (Joe Ingles: 114 min, Oscar Tshiebwe: 255 min), which may be **misleading** in practice.
- **Shai Gilgeous-Alexander** is selected in **both** versions: he is simultaneously **extreme and representative**.
- $\lambda \geq 0$ controls the trade-off: higher λ favors **representativeness**, lower λ favors **extremeness**.

ADA results: Standard vs. Penalized (III)



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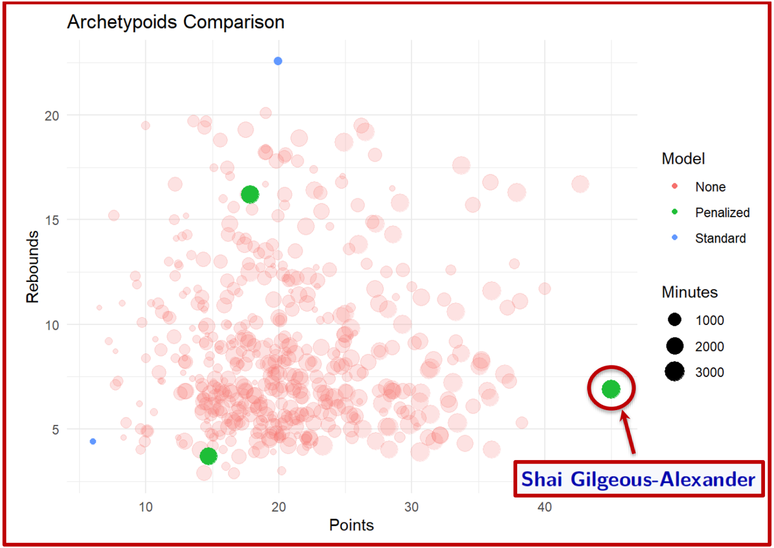

Penalized ADA results (players)

- Each NBA player is represented as a mixture of the 3 archetypoids.
  Example Cluster 3:

Cluster 3 (N=58; 11.6%)

Representative: Shai Gilgeous-Alexander

- Point guard - Shooting guard.
- 4-time NBA All-Star.
- 3-time All-NBA First Team member.
- 2-time NBA MVP.

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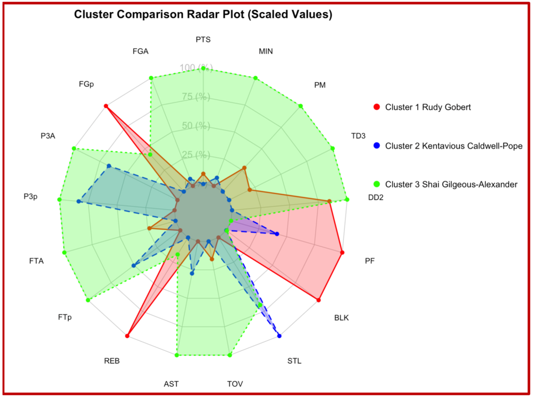

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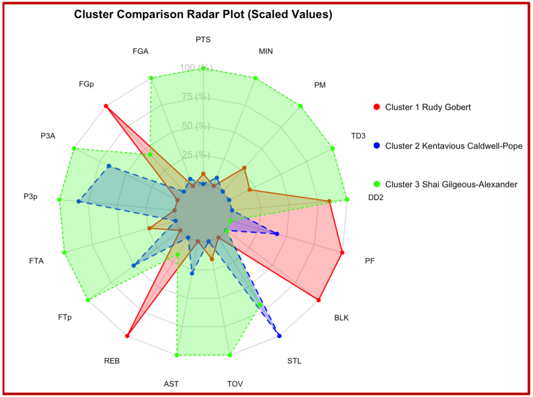

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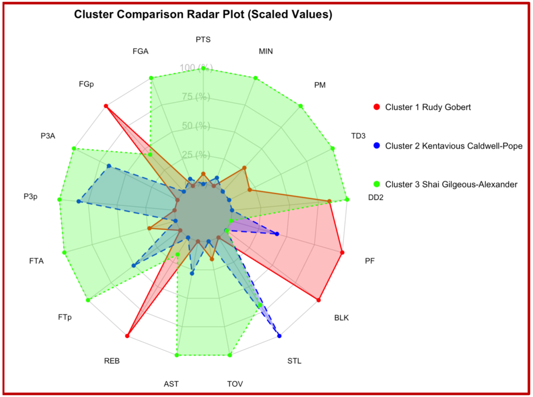

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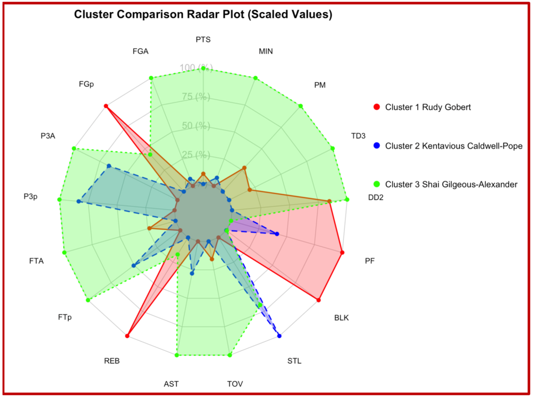

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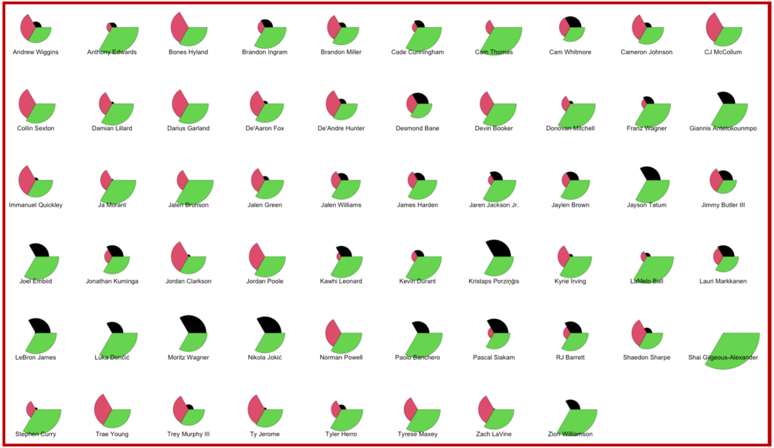
- They are the only cluster with a positive plus/minus (+1.37), confirming their **winning impact**.
- Though their heavy playmaking responsibility comes at the cost of the **highest turnover rate** (3.77).

Penalized ADA results (players)

Shai Gilgeous-Alexander (green archetypoid)



Rudy Gobert, and Kentavious Caldwell-Pope

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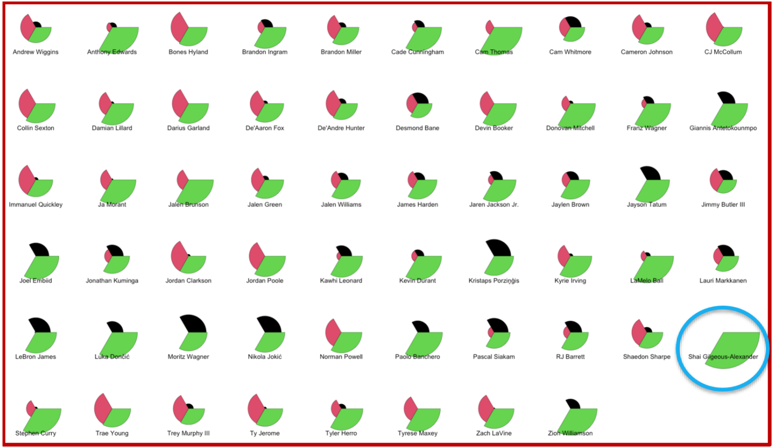

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- Standard ADA may select **outliers** with few minutes played, leading to **misleading** prototypes.
- The penalized version tends to select players with **higher minutes**, improving **interpretability** and practical relevance.
- In **sports analytics**, where real-world relevance is essential, penalized ADA provides a more **actionable** solution than standard ADA.

Main References of this Talk

- Cutler, A., and Breiman, L. (1994). Archetypal analysis. *Technometrics*, 36(4), 338-347.
- Eugster, M., and Leisch, F. (2009). From spider-man to hero-archetypal analysis in R. *Journal of Statistical Software*, 30(8), 1–23.
- Fernández, D., Epifanio, I., and McMillan, L. F. (2021). Archetypal analysis for ordinal data. *Information Sciences*, 579, 281-292.
- Vinúe, G., Epifanio, I., and Alemany, S. (2015). Archetypoids: A new approach to define representative archetypal data. *Computational Statistics & Data Analysis*, 87, 102-115.
- Vinúe, G., and Epifanio, I. (2021). Robust archetypoids for anomaly detection in big functional data. *Advances in Data Analysis and Classification*, 15, 437-462.

The End

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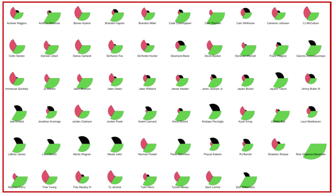

Influence of the Penalty (λ)

- For fixed $k = 3$, we compute the **average minutes** of the selected archetypoids across a range of λ values.

- As λ increases:
  1. **Avg. minutes** $\uparrow \rightarrow$ more representative players.
  2. **RSS** $\uparrow \rightarrow$ less extreme archetypoids.
  3. **Ratio** (min/RSS) $\uparrow \rightarrow$ better balance.

- Large λ favors **high-minute** players at the cost of a higher RSS.

Results for varying λ ($k = 3$)

λ	Avg. Min	RSS	Ratio
0.00	1238	8.69	142
0.10	1589	8.72	182
0.25	1589	8.72	182
0.50	1589	8.72	182
0.75	1589	8.72	182
1.00	1589	8.72	182
2.50	2063	8.92	231
5.00	2241	9.06	247
10.00	2603	9.92	262